

4th UNIT PDA

Date ___/___/___

Saathi

Check given Language is CFL OR NOT:-

- (1) $L = \{a^n b^n c^m \mid n > m\}$
- (2) $L = \{a^{m+n} b^n c^n \mid m, n \geq 1\} \rightarrow \text{DCFL, CFL}$
- (3) $L = \{a^m b^m c^n d^n \mid m, n \geq 1\} \rightarrow \text{DEFL, CFL}$
- (4) $L = \{a^m b^n c^m d^n \mid m, n \geq 1\} \rightarrow \text{not DCFL, not CFL}$
- (5) $L = \{a^m b^i c^m d^j \mid m, i, j \geq 1\} \rightarrow \text{CFL, DCFL}$
- (6) $L = \{a^i b^j c^k \mid k = i + j\} \rightarrow \text{DCFL, CFL}$
- (7) $L = \{a^i b^j c^k d^l \mid i = k, j = l\} \rightarrow \text{not DCFL, not CFL}$
- (8) $L = \{a^n b^{2n} \mid n \geq 1\} \rightarrow \text{DCFL, CFL}$
- (9) $L = \{a^i b^j c^k d^l \mid i, j, k, l \geq 1\} \rightarrow \text{DCFL, CFL}$
- (10) $L = \{a^n b^n c^n \mid n \geq 1\} \rightarrow \text{not DCFL; not CFL}$
- (11) $L = \{a^n b^n c^n d^n \mid n \geq 1\} \rightarrow \text{not DCFL, not CFL}$
- (12) $L = \{wcw^n \mid w \in \{a, b\}^+\} \rightarrow \text{DCFL, CFL}$
- (13) $L = \{ww^n \mid w \in \{a, b\}^+\} \rightarrow \text{not DCFL, CFL only}$
- (14) $L = \{ww \mid w \in \{a, b\}^+\} \rightarrow \text{not DCFL, not CFL}$
- (15) $L = \{a^{n^2} \mid n \geq 1\} \Rightarrow a, a^4, a^9, a^{16} \dots$ not in AP so not DCFL
- (16) $L = \{a^{2^n} \mid n \geq 1\} \Rightarrow a^2, a^4, a^8, \dots$ not in AP so not D, C
- (17) $L = \{a^n b^{n^2} \mid n \geq 1\} \rightarrow \text{not DCFL, not CFL}$
- (18) $L = \{a^n b^{2^n} \mid n \geq 1\} \rightarrow \text{not DCFL, not CFL}$
- (19) $L = \{a^n b^{2^n} c^{3^n} \mid n \geq 1\} \rightarrow \text{not DCFL, not CFL}$
- (20) $L = \{a^n b^n c^n \mid n \geq 1\} \rightarrow \text{not DCFL, not CFL}$

Ques: Check give Language CFL or not $L = a^p \mid p \text{ is prime}$

(Saathi)

Pumping Lemma [For Context Free Language]

* Pumping Lemma [for CFL] is used to prove that a language is NOT context free

Context Free Language:-

In formal Language theory a context free language is a language generated by some context free grammar.

The set of all CFL is identical to the set of languages accepted by pushdown Automata

context free Grammar is identified by 4 tuples as

$$G = \{V, \Sigma, S, P\} \text{ where}$$

V = set of variables or Non-terminal

Σ = set of Terminal symbol

S = start symbol

P = Production Rule

context free grammar has production Rule of the form.

$$A \rightarrow a$$

$$\text{where, } a \in \{V \cup \Sigma\}^* \quad \& \quad A \in V$$

* If A is a Context free Language, then, A has a Pumping Length " p " such that any string ' S ' where $|S| \geq p$ may be divided into 5 pieces $S = uvxyz$ such that the following conditions

conditions must be true:

- (1) $uv^i xy^i z$ is in A for every $i \geq 0$
- (2) $|vxy| > 0$
- (3) $|vxy| \leq p$

To prove that a Language is Not context free using Pumping Lemma [For CFL] follow the steps given below:-

[We Prove using CONTRADICTION]

- ⇒ Assume that A is context free
- ⇒ It has to have a Pumping Length (say p)
- ⇒ All strings longer than p can be pumped $|S| \geq p$
- ⇒ Now find a string ' S ' in such that $|S| \geq p$
- ⇒ Divide S into 5 parts $uvxyz$
- ⇒ Show that $uv^i xy^i z \notin A$ for some i
- ⇒ Then consider the ways that S can be divided into $uvxyz$.
- ⇒ Show that none of these can satisfy all the 3 pumping condition at the same time.
- ⇒ S cannot be pumped == CONTRADICTION

Ques:- show that $L = \{a^n b^n c^n \mid n \geq 0\}$ is context free.

take a string S such that $S = a^p b^p c^p$
eg:- $p=4$

$$S = a^4 b^4 c^4$$

case 1:- v & y each contain only one type of

$L = \underline{aaaa} \underline{bbbb} \underline{cccc}$
 $\begin{array}{c} \downarrow \quad \quad \quad \downarrow \\ v \quad \quad \quad x \quad \quad \quad y \quad \quad \quad z \end{array}$

(2) $|vxy| > 0$, $|aac| = 3$ Satisfy

(3) $|vxy| \leq p$ $[aaaaabbbbcc \quad |9| \leq 4]$ not satisfy

(1) $uv^i xy^i z$ $i=2$

$uv^2 xy^2 z$
 $a(aa)^2 abbbb c(c)^2 cc$
 $aaaaabbbbcccc$
 $a^6 b^4 c^5 \neq L$

Case 2:- Either v or y has more than one kind of symbol.

$L = \underline{aaaa} \underline{bbbb} \underline{cccc}$
 $\begin{array}{c} \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ v \quad \quad \quad xy \quad \quad \quad z \end{array}$

(i) $|aabb| = 5$

(ii) $|aabb| \leq 4$

(iii) $uv^i xy^i z = aa(aabb)^2 bcccc$

$\Rightarrow aa aabbaabbbbcccc \notin L$

$\Rightarrow a^4 b^2 a^2 b^3 c^4$ not CFG

Que Show that $L = \{ ww \mid w \in \{0,1\}^* \}$ is Not Context free.

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- $\Rightarrow$ Assume that L is Context free
- $\Rightarrow$ L must have a pumping length (say P)
- $\Rightarrow$ Now we take a string S such that

$$S = 0^P 1^P 0^P 1^P$$
- $\Rightarrow$ We divide S into parts $uvxyz$

Eg $P=5$ ϕ

$$S = 0^5 1^5 0^5 1^5$$

CASE 1:- vxy does not straddle a boundary

$$\underbrace{00000}_u \underbrace{1111}_{vxy} \underbrace{00000}_z \underbrace{11111}_z$$

$$\begin{aligned} |vxy| &\leq P & |1111| &> 0 \quad \checkmark \\ |vxy| &\leq P & |1111| &\Rightarrow |3| \leq |5| \text{ which } P \text{ Length} \end{aligned}$$

$$uv^i xy^i z$$

$$i=2$$

$$uv^2 xy^2 z$$

$$00000 [1]^2 [1]^2 00000 11111$$

$$00000 11111 1100000 11111$$

$$0^5 1^7 0^5 1^5$$

$$\underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$$

Here 1st half not equal to 2nd half.

do

$$\boxed{0^5 1^7 0^5 1^5 \notin L}$$

CASE 2a: vxy straddles the first boundary

00000'11111'00000'11111
 $\underbrace{\hspace{1cm}}_u \underbrace{\hspace{1cm}}_{vxy} \underbrace{\hspace{1cm}}_z$

$$|vxy| > 0$$

$$|vxy| \leq p \quad |5| \leq 5$$

$$uv^i xy^i z \Rightarrow uv^2 xy^2 z$$

$i=2$

000 [00]² 1 [11]² 1 00000 11111
 0000000 11111 100000 111
 $0^7 1^7 0^5 1^5$
 $\underbrace{\hspace{1cm}} \underbrace{\hspace{1cm}}$

So 1st half not equal to 2nd Half.

$$0^7 1^7 0^5 1^5 \notin L$$

CASE 2b: vxy straddles the third boundary

00000'11111'00000'11111
 $\underbrace{\hspace{1cm}}_u \underbrace{\hspace{1cm}}_{vxy} \underbrace{\hspace{1cm}}_z$

$$uv^2 xy^2 z$$

00000 11111 0000000 11111
 $0^5 1^5 0^7 1^7$
 $\underbrace{\hspace{1cm}} \underbrace{\hspace{1cm}}$

So 1st half not equal to 2nd Half.

$$0^5 1^5 0^7 1^7 \notin L$$

CASE 3 & 4:- VXY straddles the midpoint.

00000'11111'00000'11111
 $\underbrace{\hspace{1.5cm}}_U \quad \underbrace{\hspace{1.5cm}}_{VXY} \quad \underbrace{\hspace{1.5cm}}_Z$

UV^2XY^2Z

00000111111000000011111

$\underbrace{0^5}_U \underbrace{1^7 0^7 0^5}_{VXY}$

1st half not equal to 2nd half

$0^5 1^7 0^7 0^5 \notin L$

In all 4 cases our condition not satisfy
 so our language is not
 context free

PUSHDOWN AUTOMATA

A PDA is a way to implement a Context free grammar in a similar way we design finite automata for regular grammar.

- * It is powerful than FSM
- * FSM has a very limited memory but PDA has more memory.
- * PDA = Finite State Machine + A Stack.

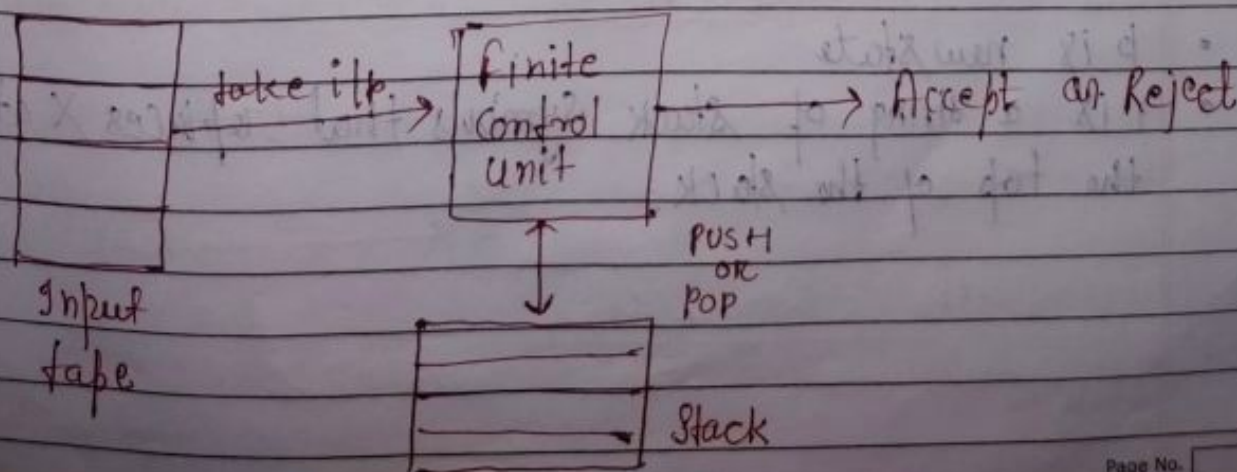
Stack:- It is a data structure, a stack is a way we arrange elements one on top of another.

A stack does two basic operations.

- PUSH:- A new element is added at the Top of the stack
- POP:- The top element of the stack is read & removed.

Pushdown Automata has 3 components:-

- ① Input tape
- ② A finite control
- ③ A stack with infinite size.



PDA:

A PDA is formally defined by 7 tuple as shown.

$$P = (Q, \Sigma, \Gamma, \delta; q_0, Z_0; F)$$

Where,

- $Q \Rightarrow$ A finite set of symbols
- $\Sigma \Rightarrow$ A finite set of input symbols
- $\Gamma \Rightarrow$ A finite stack alphabet
- $\delta \Rightarrow$ The Transition Function
- $q_0 \Rightarrow$ The start state
- $Z_0 \Rightarrow$ The start stack symbol
- $F \Rightarrow$ The set of final / Accepting states

δ takes as argument a triple $\delta(q, a, X)$ where:

- (i) q is a state in Q
- (ii) a is either an input symbol in Σ or $a = \epsilon$
- (iii) X is a stack symbol, that is a member of Γ

The output of δ is finite set of pairs (p, Y) where:

- p is new state
- Y is a string of stack symbols that replaces X at the top of the stack.

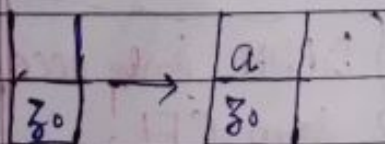
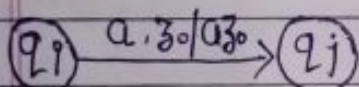
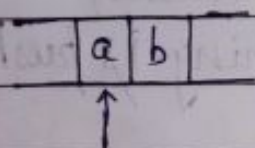
eg: If $Y = \epsilon$ then the stack is popped

If $Y = X$ then the stack is unchanged

If $Y = YZ$ then X is replaced by Z and Y is pushed onto the stack

* Basic operation on stacks:

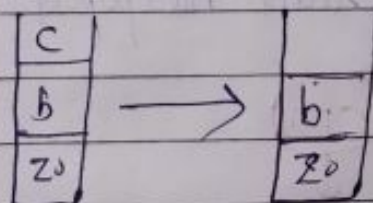
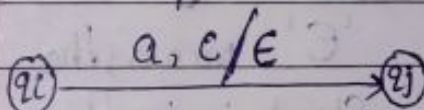
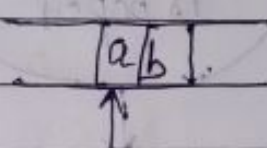
(1) PUSH



Stack

$$\delta(q_i, a, z_0) \rightarrow (q_j, a z_0)$$

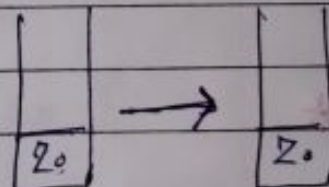
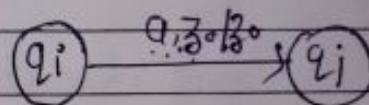
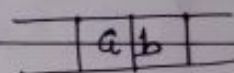
(2) POP



$$\delta(q_i, a, c) \rightarrow (q_j, \epsilon)$$

(3) SKIP

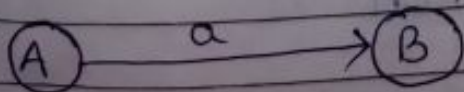
(no push & no pop)



$$\delta(q_i, a, z_0) \rightarrow (q_j, z_0)$$

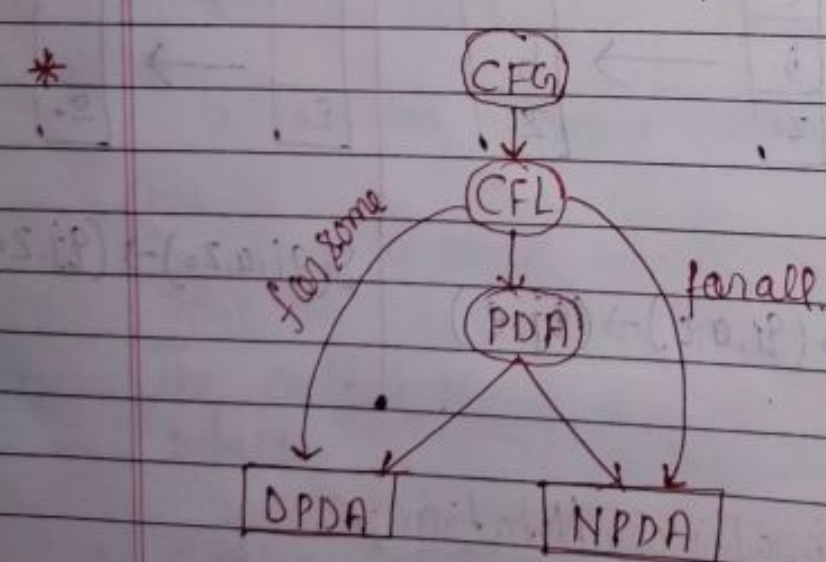
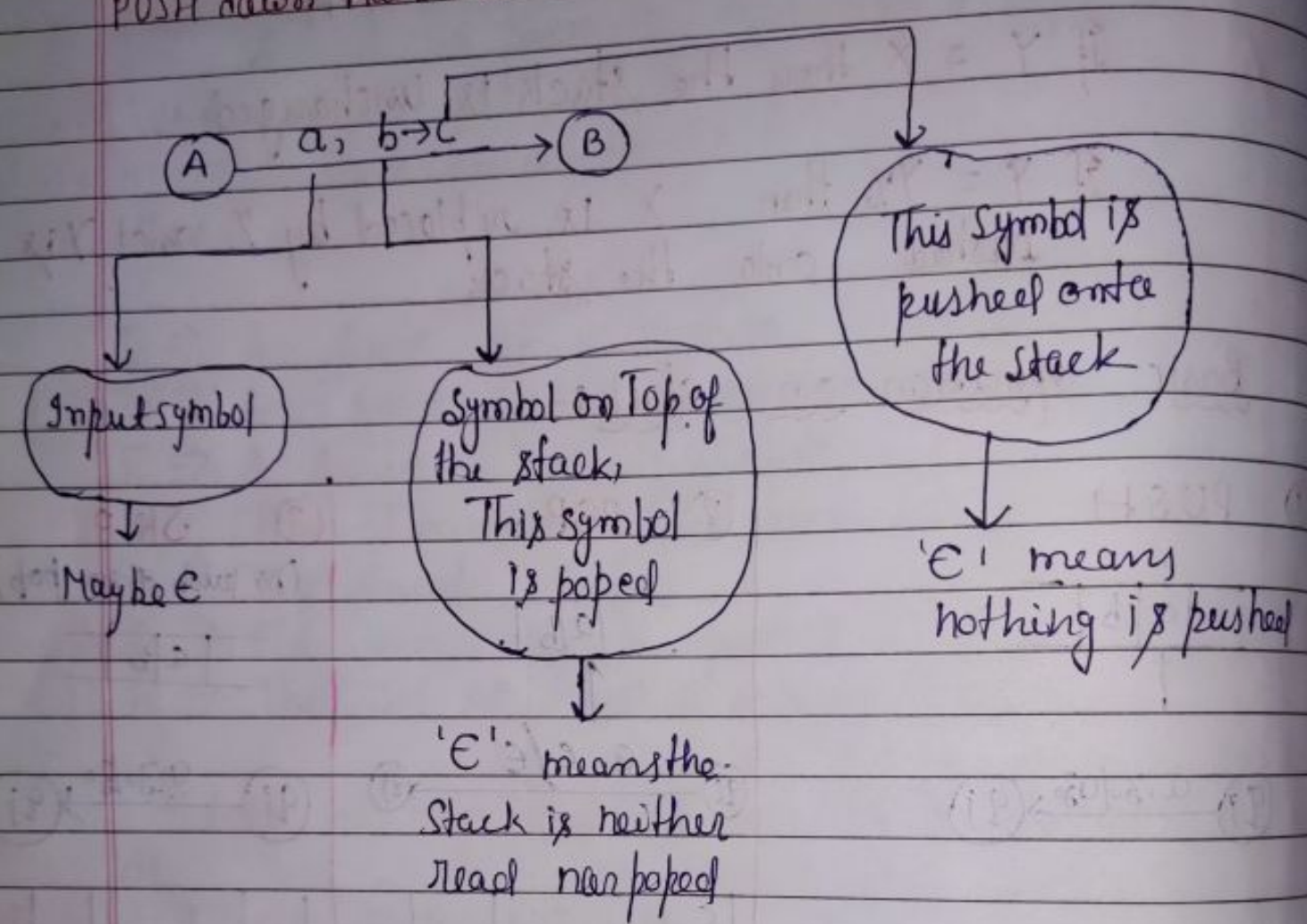
PUSHDOWN Automata { Graphical: Notation }

Finite state Machine



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PUSH down Automata



DPDA only accept some CFL, which is known as Deterministic CFL, [DCFL]

NPDA accepts all CFL

DPDA $\rightarrow$ Deterministic Pushdown Automaton
 NPDA $\rightarrow$ Non-deterministic PDA

$P(NPDA) > P(DPDA)$
 NPDA is more powerful than DPDA

Transition function of: DPDA & NPDA...

DPDA:-

$$\delta : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$$

NPDA:-

$$\delta : Q \times \Sigma \times \Gamma \rightarrow \{Q \times \Gamma^*\}$$

Eg:- Let PDA $A = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$
 $Q = \{q_0, q_1, q_f\}$ $\Sigma = \{a, b\}$ $\Gamma = \{a, Z_0\}$ $F = \{q_f\}$
 $\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$
 $\delta(q_0, a, a) = \{(q_0, aa)\}$
 $\delta(q_0, b, a) = \{(q_1, \epsilon)\}$
 $\delta(q_0, b, Z_0) = \{(q_1, \epsilon)\}$
 $\delta(q_1, \epsilon, Z_0) = \{(q_f, \epsilon)\}$

input string
 $w = aabb$

Soln:

$\delta(q_0, aabb, Z_0)$	(q_0, abb, aZ_0)
	(q_0, bb, aaZ_0)
	(q_1, b, aZ_0)
	(q_1, ϵ, Z_0)
	$(q_f, \epsilon, \epsilon)$

(String accepted)

> String accepted case:-

Eg:-

① if

In this example we are on initial state but stack is empty.

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Eg: Let input string $aaabbb$

$$\delta(q_0, a, Z_0) = (q_0, aZ_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \Lambda)$$

$$\delta(q_1, b, a) = (q_1, \Lambda)$$

$$\delta(q_1, \Lambda, Z_0) = (q_f, Z_0)$$

$$\rightarrow (q_0, aaabbb, Z_0) \vdash (q_0, aaabbb, qZ_0)$$

$$\vdash (q_0, a bbb, aaZ_0)$$

$$\vdash (q_0, bbb, \Lambda aaZ_0)$$

$$\vdash (q_1, bb, \Lambda aZ_0)$$

$$\vdash (q_1, b, \Lambda Z_0)$$

$$\vdash (q_1, \Lambda, Z_0)$$

$$\vdash (q_f, \Lambda, Z_0)$$

In this example we are on final state but stack is non empty.

PDA Acceptance string:

⇒ String is accepted by the PDA when we get to the acceptance state and see that the stack is empty.

•) PDA can accept by accepting both empty state & empty stack.

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⇒ Graphical representation of PDA

Eg:-

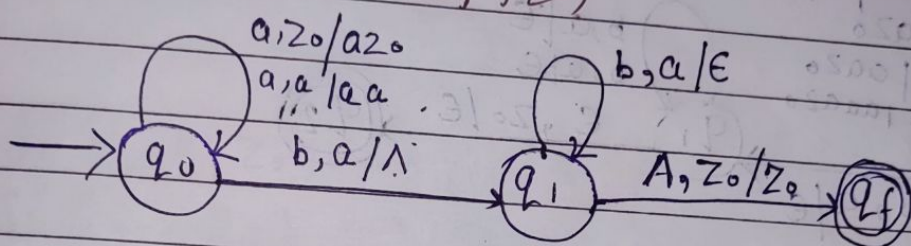
$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \Lambda)$$

$$\delta(q_1, b, a) = (q_1, \Lambda)$$

$$\delta(q_1, \Lambda, z_0) = (q_f, z_0)$$



Ques Construct the PDA for the language $L = \{a^n b^n \mid n \geq 1\}$

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, f\}$$

Step 1:-

Initially push all a's onto the stack

Step 2:-

Whenever "b" occurs change the state & pop "a" from the stack.

Step 3:-

Repeat step 2 until stack is empty.

$$L = \{ab, aabb, aaabbb, \dots\}$$

a	a	a	b	b	b	ε
---	---	---	---	---	---	---

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aaz_0)$$

$$\delta(q_0, a, a) = (q_0, aaaz_0)$$

$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

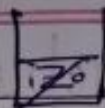
$$\delta(q_1, b, a) = (q_1, \epsilon)$$

ε
ε
ε
z0

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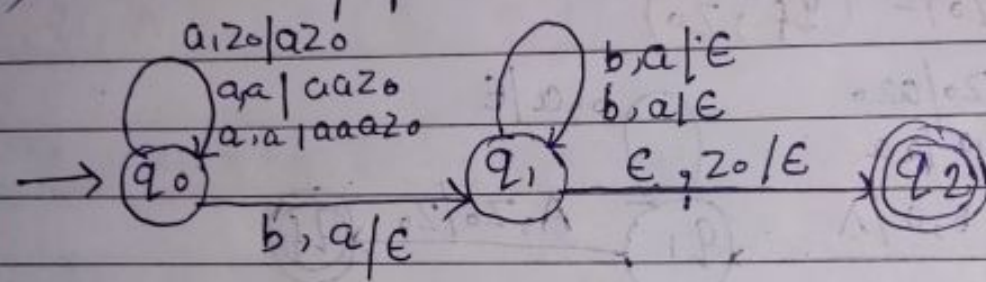
Saa

$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$



Here input string completely ended, so we will change the state.

z_0 also pop, now stack become empty.



Now check whether this string accept or not
 $w = aabb$

$\delta(q_0, aabbz_0)$	(q_0, abb, az_0)
	$(q_0, bb, aa z_0)$
	$(q_1, b, a z_0)$
	(q_1, ϵ, z_0)
	(q_2, ϵ)

final state & stack is empty.

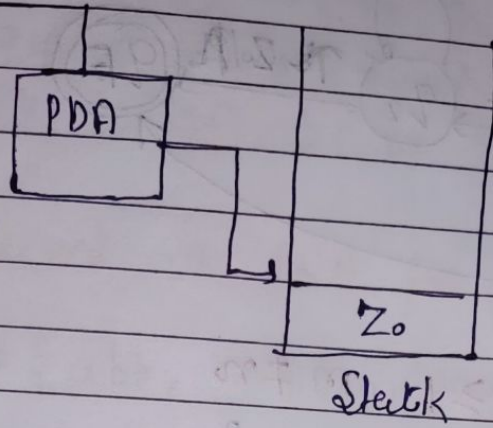
Design PDA for

- (i) $L = a^n b^n \mid n \geq 0$
- (ii) $L = a^n b^{2n} \mid n \geq 0$
- (iii) $L = a^n b^n c^m \mid n, m \geq 1, m \neq n$
- (iv) $L = a^m b^n c^n \mid n, m \geq 1, m \neq n$
- (v) $L = a^n b^n \mid n \geq 0$
 $L = \{ \epsilon, ab, aabb, aaabbb, \dots \}$

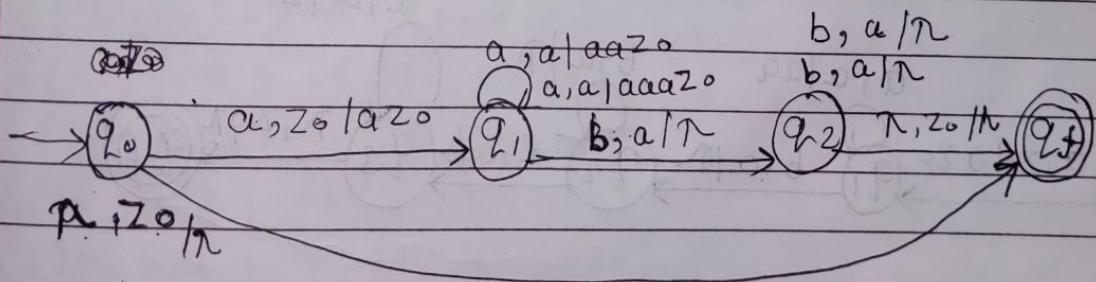
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a a a b b b ϵ i/p tape



- $\delta(q_0, \epsilon, Z_0) \rightarrow (q_f, \epsilon)$
- $\delta(q_0, a, Z_0) \rightarrow (q_1, aZ_0)$
- $\delta(q_1, a, a) \rightarrow (q_1, aaZ_0)$
- $\delta(q_1, a, a) \rightarrow (q_1, aaaZ_0)$
- $\delta(q_1, b, a) \rightarrow (q_2, \epsilon)$
- $\delta(q_2, b, a) \rightarrow (q_2, \epsilon)$
- $\delta(q_2, b, a) \rightarrow (q_2, \epsilon)$
- $\delta(q_2, \epsilon, Z_0) \rightarrow (q_f, \epsilon)$

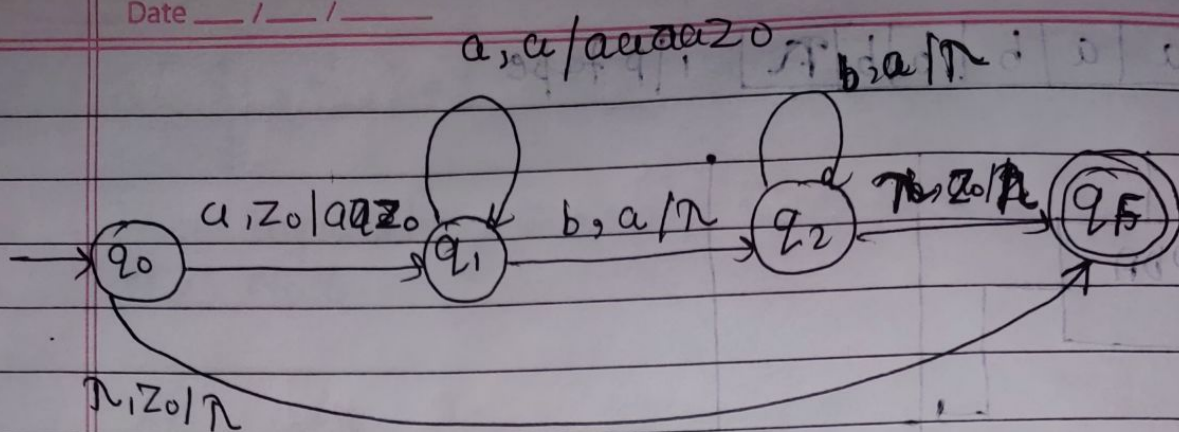


(ii) $L = a^n b^{2n} \mid n \geq 0 \Rightarrow L = \{ \epsilon, abb, aa bbbb, aaa bbbbbb, \dots \}$

- $\delta(q_0, \epsilon, Z_0) \rightarrow (q_f, \epsilon)$
- $\delta(q_0, a, Z_0) \rightarrow (q_1, aZ_0)$
- $\delta(q_1, a, a) \rightarrow (q_1, aaZ_0)$
- $\delta(q_1, b, a) \rightarrow (q_2, \epsilon)$
- $\delta(q_2, b, a) \rightarrow (q_2, \epsilon)$
- $\delta(q_2, \epsilon, Z_0) \rightarrow (q_f, \epsilon)$

for every a we insert two a's into stack

a
a
a
a
Z0



(iii) $L = \{a^n b^n c^m \mid n, m \geq 1, m \neq n\}$
 $L = \{abcc, aabccc, \dots\}$

$\delta(q_0, a, z_0) \rightarrow (q_1, az_0)$

$\delta(q_1, a, a) \rightarrow (q_1, aa)$

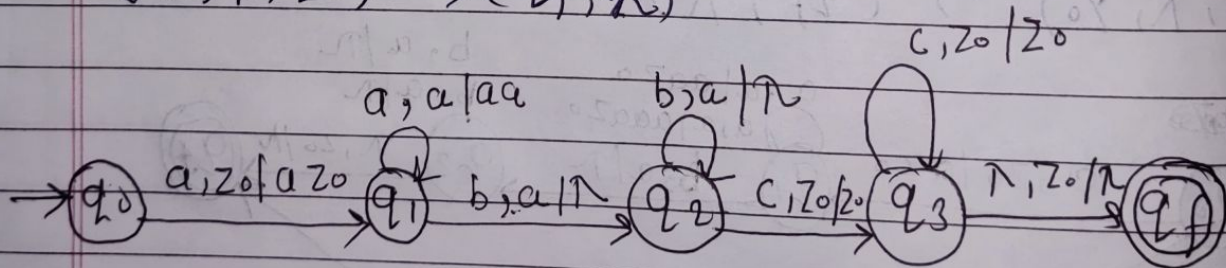
$\delta(q_1, b, a) \rightarrow (q_2, \lambda)$

$\delta(q_2, b, a) \rightarrow (q_2, \lambda)$

$\delta(q_2, c, z_0) \rightarrow (q_3, z_0)$

$\delta(q_3, c, z_0) \rightarrow (q_3, z_0)$

$\delta(q_3, \lambda, z_0) \rightarrow (q_f, \lambda)$



(iv) $L = \{a^m b^n c^n \mid n, m \geq 1, m \neq n\}$
 $L = \{abbcc, aabbbccc, aaabbbbbbcccc, \dots\}$

$\delta(q_0, a, z_0) \rightarrow (q_1, z_0)$

$\delta(q_1, a, z_0) \rightarrow (q_1, z_0)$

$\delta(q_1, b, z_0) \rightarrow (q_2, bz_0)$

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$$\delta(q_2, b, b) \rightarrow (q_2, bb)$$

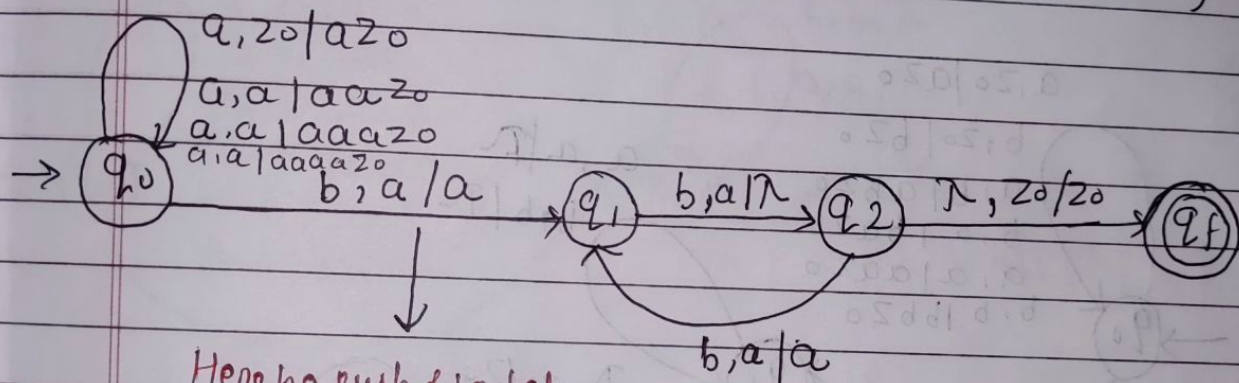
$$\delta(q_3, c, b) \rightarrow (q_4, \tau)$$

$$\delta(q_4, c, b) \rightarrow (q_4, \tau)$$

$$\delta(q_4, \tau, z_0) \rightarrow (q_f, \tau)$$

Ques construct a PDA for $L = a^n b^{2n} \mid n \geq 1$

$$L = \{ abb, aabb, aaabbb, aaaaabbbb, \dots \}$$



Here no push & no pop operation will perform

$$\delta(q_0, a, z_0) \rightarrow (q_0, a z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, a a z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, a a a z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, a a a a z_0)$$

$$\delta(q_0, b, a) \rightarrow (q_1, a a a a z_0)$$

$$\delta(q_1, b, a) \rightarrow (q_2, \tau)$$

$$\delta(q_2, b, a) \rightarrow (q_1, a a a z_0)$$

$$\delta(q_2, \tau, z_0) \rightarrow (q_f, z_0) \quad \text{accepted}$$

Ques construct PDA for the given $L = (w c w^r \mid w \in (a, b)^*)$

$$w = ab \quad w^r = ba \quad w c w^r = abcba$$

$$w = zbaba \quad w^r = abab \quad w c w^r = babacabab$$

$$w = a \quad w^r = a \quad w c w^r = aca$$

$$w = \epsilon \quad w^r = \epsilon \quad w c w^r = \epsilon c \epsilon = c$$

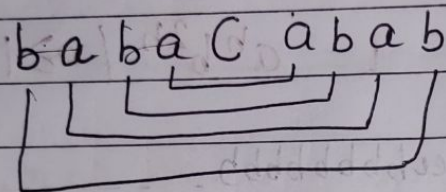
Here C will be decision making

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Date ___/___/___

Language will be

$L = \{C, aca, bcb, abcbab, babaCabab, \dots\}$



$a, z_0 / az_0$

$b, z_0 / bz_0$

$a, b / abz_0$

$b, a / baz_0$

$a, a / aaz_0$

$b, b / bbz_0$

$a, a / \pi$

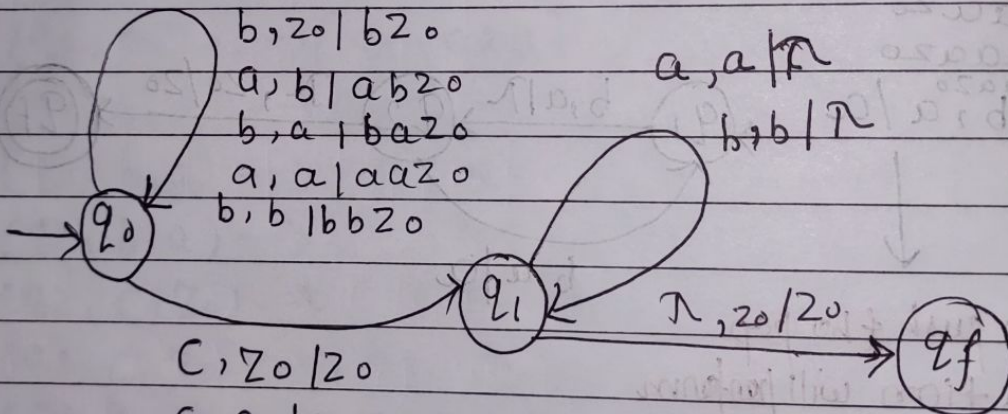
$b, b / \pi$

$\pi, z_0 / z_0$

$C, z_0 / z_0$

$C, a / a$

$C, b / b$

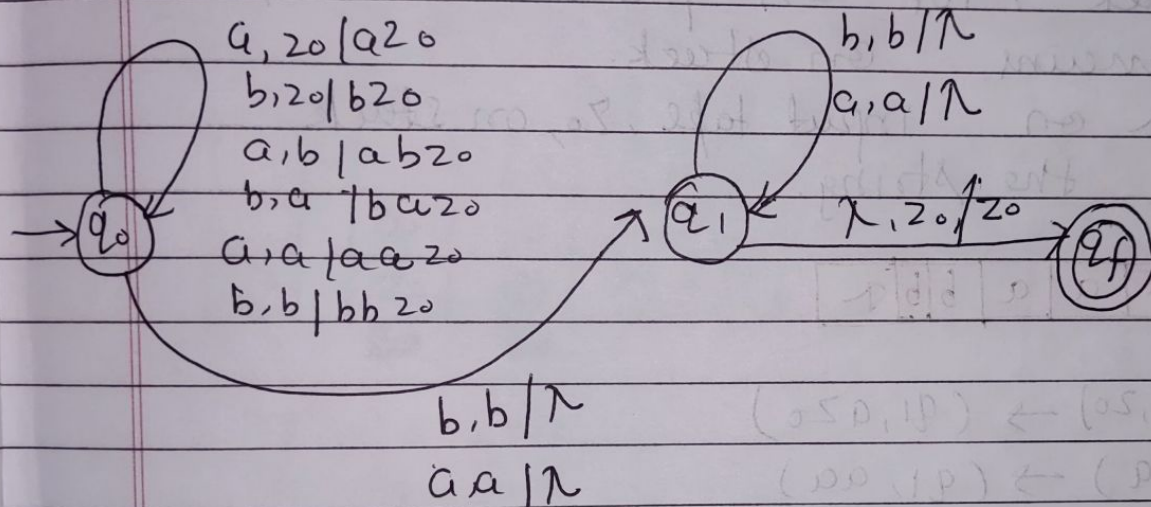


b
a
z0

a
b
z0

a'
a
z0

b
b
z0

$$\omega = a, \quad \omega^n = a, \quad \omega\omega^n = a a$$
$$w = ab \quad w^n = b a \quad w w^n = a b b a$$
$$w = abab \quad w^\eta = baba \quad ww^\eta = ababbaba$$
$$L = \{ aa, \underline{abba}, \underline{ababbaba} \dots \}$$


Ques:- Design PDA for Balanced parentheses

CLF 32)

- ⇒ 1st push C into stack
- ⇒ Now push all { into stack
- ⇒ When } comes, pop each {
- ⇒ when) comes, pop each C

1	
2	
3	
4	5
20	20

$$S(q_0, c, z_0) \rightarrow (q_1, (z_0))$$
$$\delta(q_1, \{, C\} \rightarrow \{q_2, \{C\}$$
$$\delta(q_2, \{ \}) \rightarrow (q_2, \{ \})$$
$$\delta(q_2, \gamma, f) \rightarrow (q_3, \pi)$$
$$\delta(q_2, \{, \}) \rightarrow (q_3, \wedge)$$
$$\delta(q_3, \epsilon) \rightarrow (q_4, \epsilon)$$
$$\delta(q_3, \lambda, z_0) \rightarrow (q_f, \lambda)$$

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Ques Design PDA for $L = \{a^m b^n \mid m > n \geq 1\}$

$L = \{aab, aaab, aaabbb, \dots\}$

- $\Rightarrow$ Push all a's into stack
- $\Rightarrow$ When b's comes, POP a's from stack
- $\Rightarrow$ If empty string λ comes, and a's on stack, POP a's from stack till Z_0 remains on stack.
- $\Rightarrow$ If λ on input tape, Z_0 on stack accept the string.

a	a	a	a	b	b	λ
---	---	---	---	---	---	-----------

$\delta(q_0, a, Z_0) \rightarrow (q_1, aZ_0)$

$\delta(q_1, a, a) \rightarrow (q_1, aa)$

$\delta(q_1, b, a) \rightarrow (q_2, \lambda)$

$\delta(q_2, b, a) \rightarrow (q_2, \lambda)$

$\delta(q_2, \lambda, a) \rightarrow (q_2, \lambda)$

$\delta(q_2, \lambda, Z_0) \rightarrow (q_f, \lambda)$

Ques

Design PDA $L = \{w \mid n_a(w) = n_b(w), w \in \{a,b\}^*\}$

★

$L = \{\lambda, a, b, ab, ba, aabb, bbaa, \dots\}$

$\delta(q_0, \lambda, Z_0) \rightarrow (q_f, \lambda)$

$\delta(q_0, a, Z_0) \rightarrow (q_0, aZ_0)$

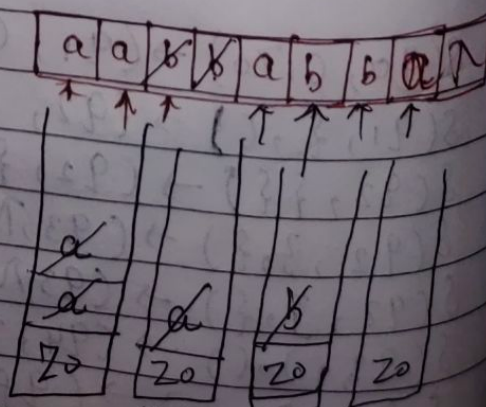
$\delta(q_0, b, Z_0) \rightarrow (q_0, bZ_0)$

$\delta(q_0, a, a) \rightarrow (q_0, aa)$

$\delta(q_0, b, b) \rightarrow (q_0, bb)$

$\delta(q_0, a, b) \rightarrow (q_0, \lambda)$

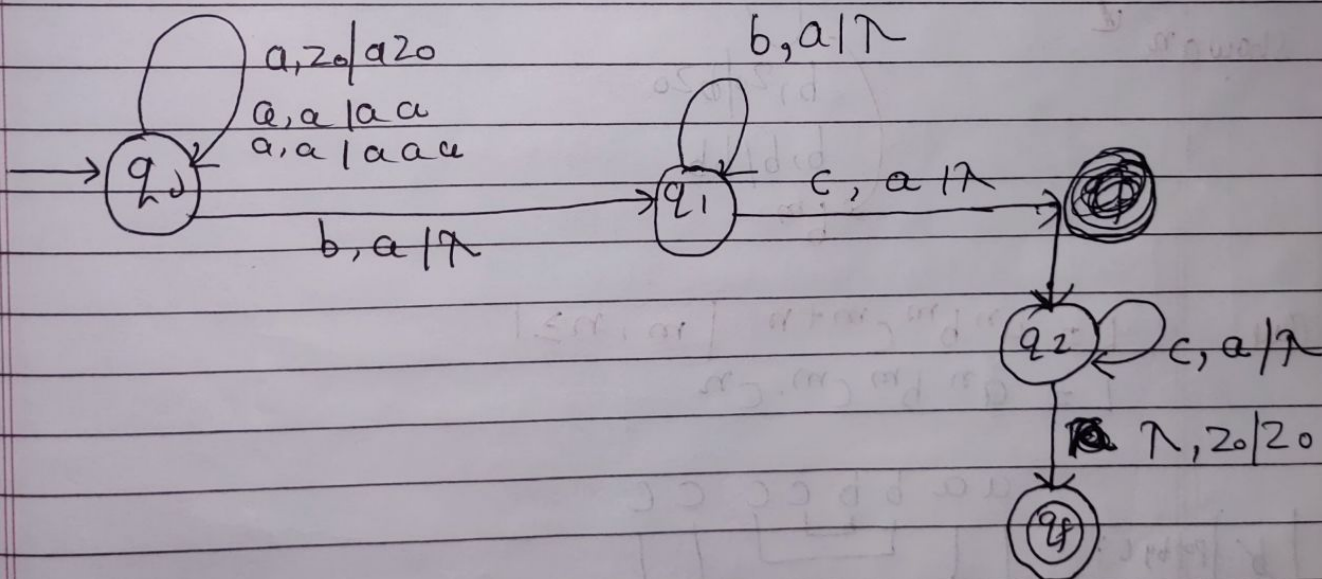
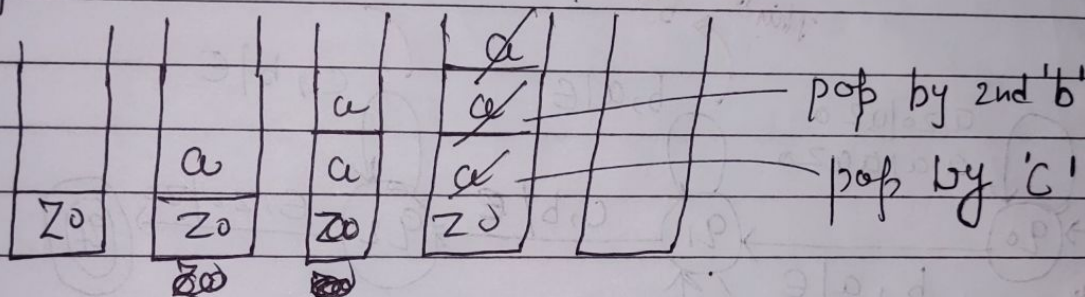
$\delta(q_0, b, a) \rightarrow (q_0, \lambda)$



Ques Construct PDA $L = a^{m+n} b^m c^n \mid m, n \geq 1$
 $\{m=1, n=1\} \{m=2, n=1\}$
 $\{ \text{zaabc}, \text{aaabbc} \dots \}$

* In this all 'a's' will be PUSH into stack & 'a' will be pop when insert 'b', & 'a' also pop when insert 'c'

Eg: a a a b b c — pop by 1st 'b'



Ques $L = a^n b^{m+n} c^m \mid m, n \geq 1$
 $= a^n b^n b^m c^m$

- all no a's PUSH into stack & Pop by all no of b's (b^n)
- then all no of b's (b^m) pop by all no of c's (c^m)

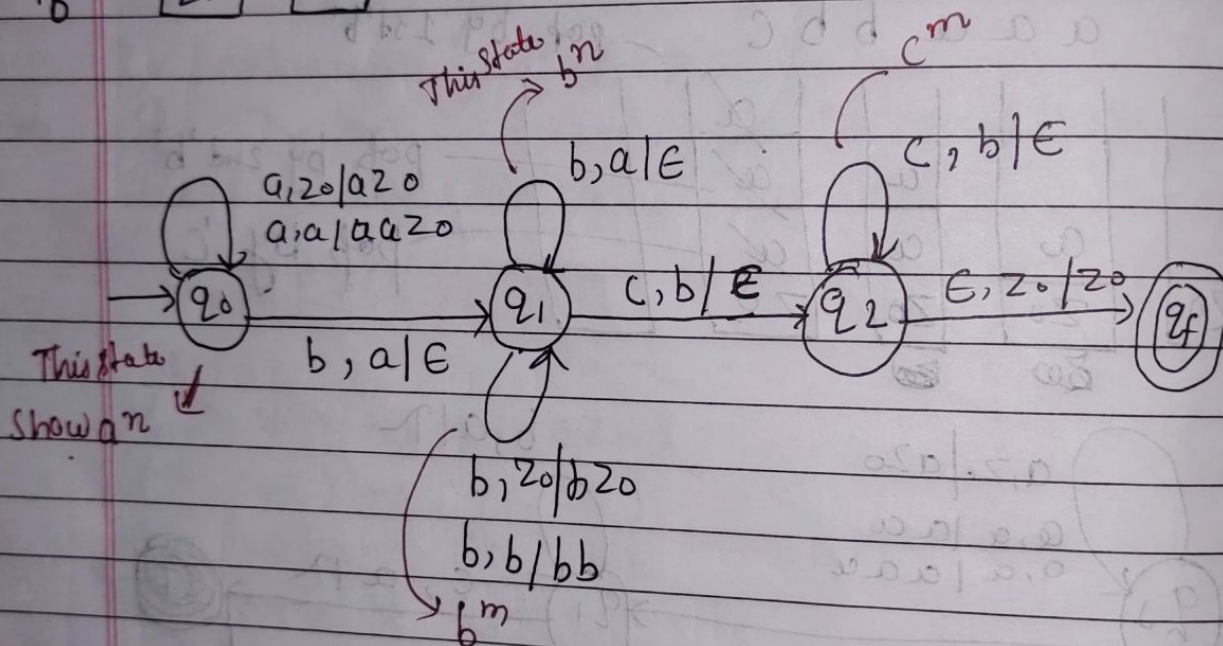
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eg $a a b b b c c$

all's
pop by
'b'

	a	b
	a	b
z0	z0	z0

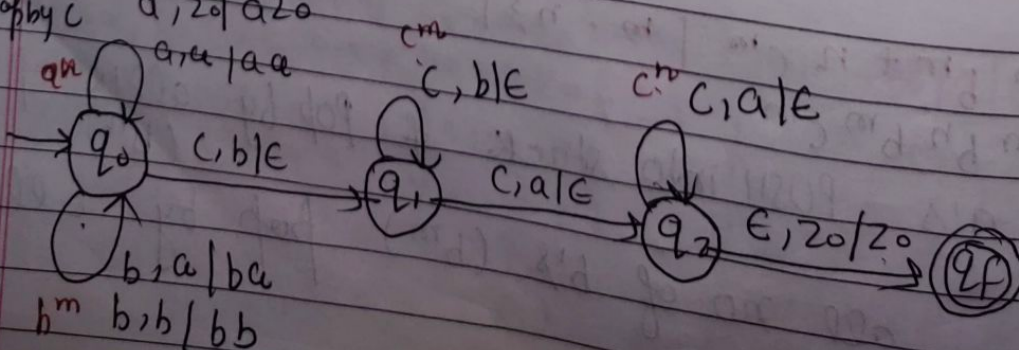
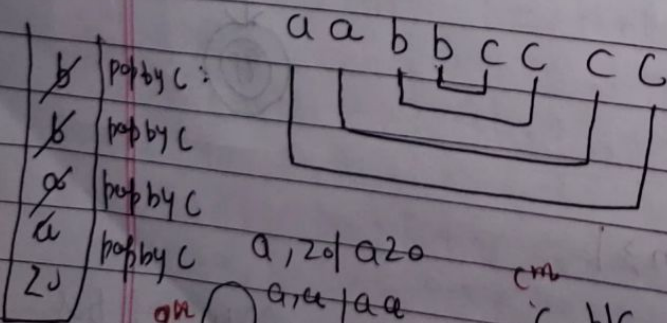
all b's pop
by c's



Ques

$$L = a^n b^m c^{m+n} \quad | \quad m, n \geq 1$$

$$L = a^n b^m c^m \cdot c^n$$

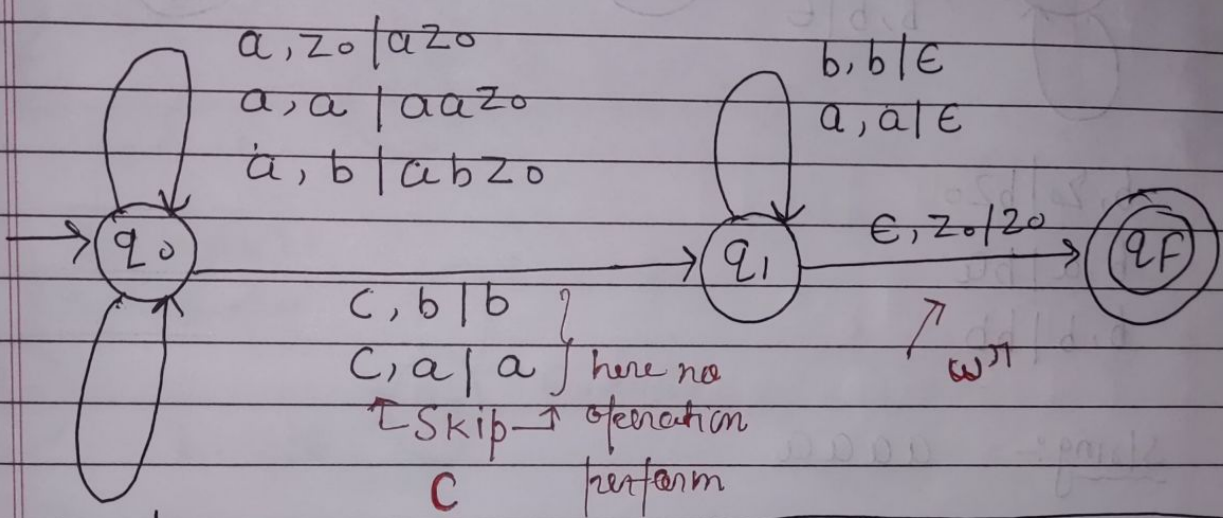




Ques

Design PDA $L = \{w c w^R \mid w \in \{a, b\}^+\}$

$\overbrace{a b b}^w \quad c \quad \overbrace{b b a}^{w^R}$
 $\overbrace{b a a}^w \quad c \quad \overbrace{a a b}^{w^R}$



$b, z_0 \mid b z_0$

$b, b \mid b b z_0$

$b, a \mid b a z_0$

all these represent w

On state q_0 , when i/p a then top of stack all are different symbols, $\{z_0, a, b\}$
 q_0 , when i/p b , then top of stack $\{z, b, a\}$
 so this is DPDA

Check string: $b a a c a a b$
 $\uparrow \uparrow \uparrow$ for c skip operation

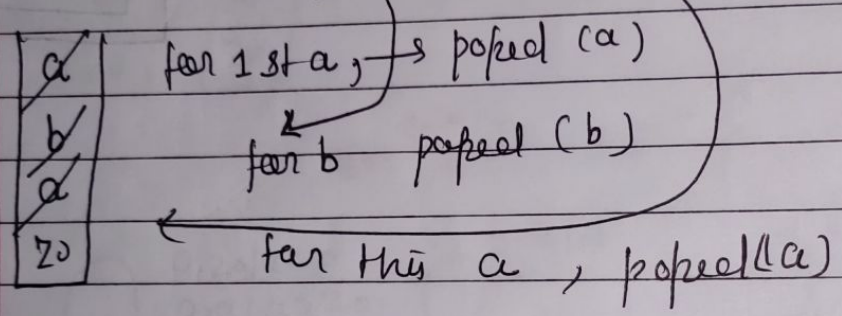
a
a
b
z_0

Date ___/___/___

Qw Design PDA for the given Language $L = ww^R$ | we have to check whether it is DPDA or NPDA.

$w = abba$ $w^R = abba$ $ww^R = abbaabba$
 $w = aa$ $w^R = aa$ $ww^R = aaaa$

⇒ String abba



⇒ String aaaa

a	
a	a
Z ₀	Z ₀

for this 'a' we have two choice
 'a' will be PUSH,
 'a' will be POP

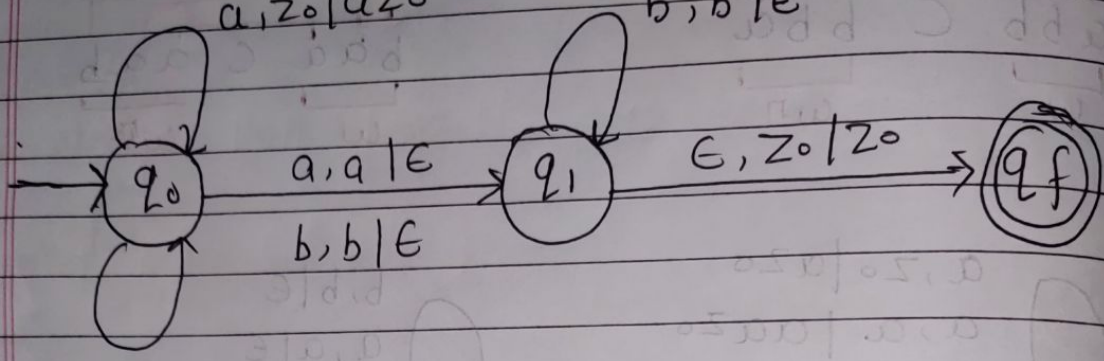
- * means when top of stack 'a' & input 'a' then we will pop.
- * on other side if 'a' & top of stack a then we will PUSH 'a'

At a time same input symbol 'a' we will perform two transition on 2 state. (operation PUSH & POP)
 Which is the case of NPDA

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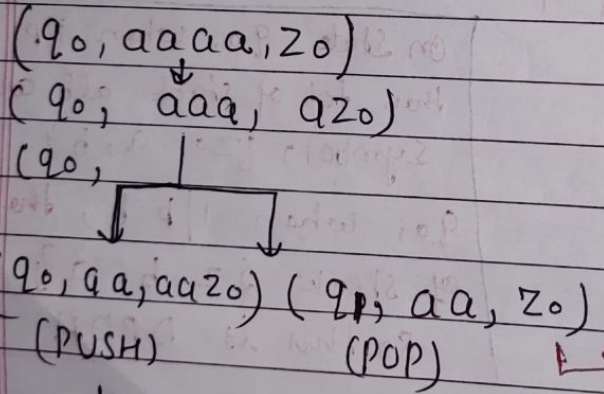
a, a | aa
a, b | ab
a, z₀ | az₀

a, a | ε
b, b | ε



b, z₀ | bz₀
b, a | ba
b, b | bb

String:- aaaa



Here this transition is not define & Stop

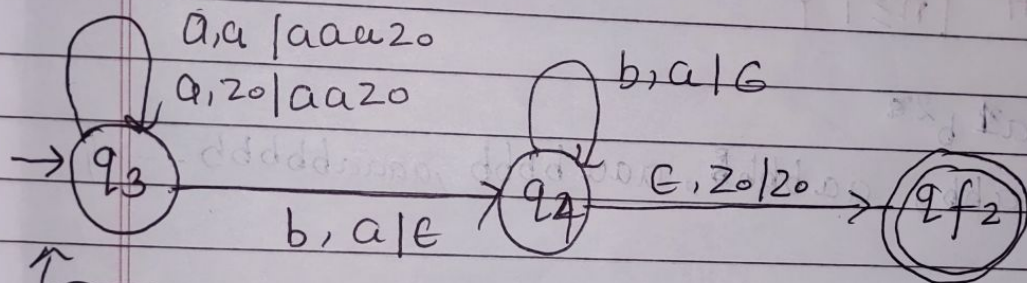
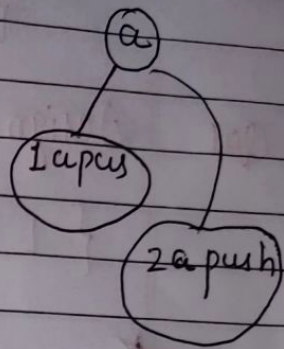
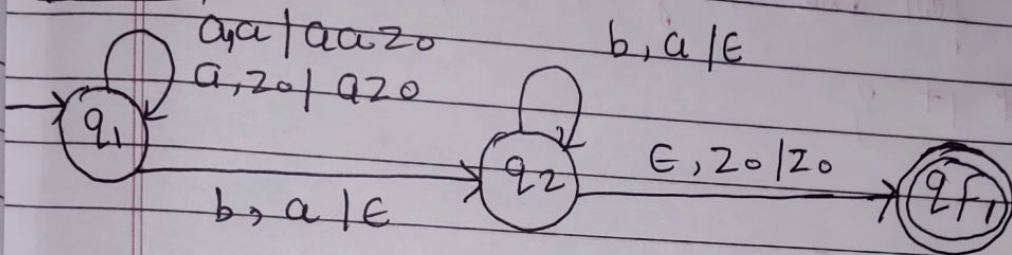
find final state.

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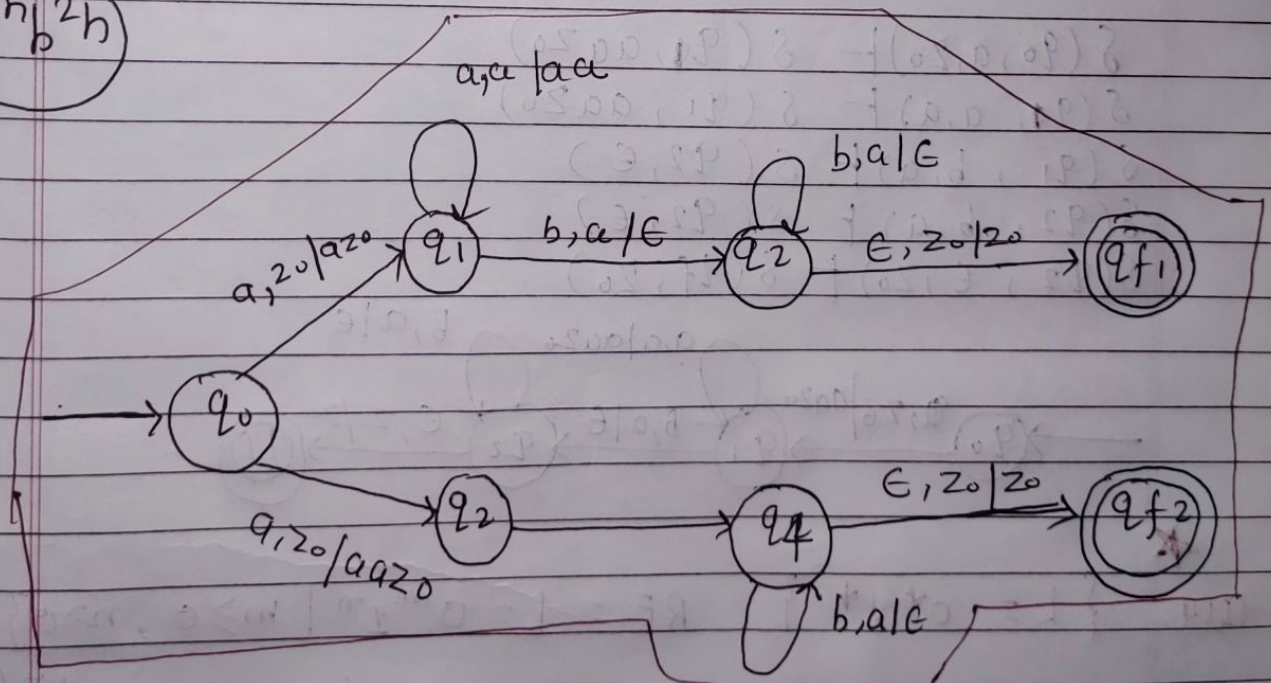
saathi

Ques Design PDA for $\{ L = a^n b^n \text{ UNION } a^n b^{2n} \mid n \geq 1 \}$
 $L = [\{ a^n b^n \} \cup \{ a^n b^{2n} \} \mid n \geq 1]$

$a^n b^n$



$a^n b^{2n}$



This is also NDPDA

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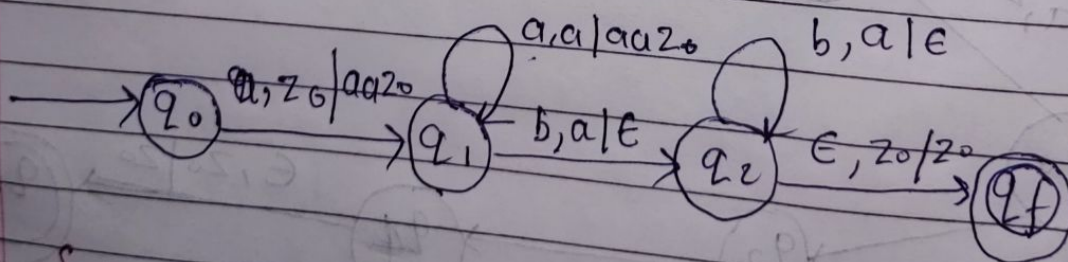
Ques Design a PDA for the Language
 $\{L = a^n b^{n+1} \mid n \geq 1\}$

Ques Design PDA for the regular Expression
 $\{L = 0^* 1^+\}$

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 Ques: $\{L = a^n b^{n+1} \mid n \geq 1\}$

$L = a^1 b^2$
 $\{L = abb, aabbb, aaabbbb, aaaabbbbb \dots\}$

$\delta(q_0, a, z_0) \vdash \delta(q_0, aa, z_0)$
 $\delta(q_0, a, a) \vdash \delta(q_1, aa, z_0)$
 $\delta(q_1, b, a) \vdash \delta(q_2, \epsilon)$
 $\delta(q_2, b, a) \vdash \delta(q_2, \epsilon)$
 $\delta(q_2, \epsilon, z_0) \vdash \delta(q_f, z_0)$



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 Ques $\{L = 0^* 1^+\}$ RE = $L = 0^m 1^n \mid m \geq 0, n \geq 1$

$\delta(q_0, 0, z_0) \vdash (q_0, 0 z_0)$
 $\delta(q_0, 1, z_0) \vdash (q_f, z_0)$
 $\delta(q_1, 1, z_0) \vdash (q_f, z_0)$

